SOLUTION I.

Let D denote the diameter of the sphere,
H altitude of the dome.
r radius of its base,
h altitude of the cone;

then $$r^2 = DH - H^2,$$

and the volume of the dome, being the difference of the volumes of the sector and the cone whose vertex is at the centre of the sphere, is

$$\tfrac{1}{6}\pi HD^2 - \tfrac{1}{6}\pi r^2(D - 2H) = \tfrac{1}{6}\pi H^2(3D - 2H).$$

The volume of the required cone is, again,

$$\tfrac{1}{3}\pi hr^2 = \tfrac{1}{3}\pi Hh(D - H).$$

The two volumes are equal by hypothesis;

$$\therefore \frac{h}{H} = \frac{3D - 2H}{2D - 2H}.$$ [*Charles Puryear.*]

SOLUTION II.

Let D_1 be the diameter of the base of the dome; the condition gives

$$\frac{\pi}{12} D_1^2 h = \frac{\pi}{8} D_1^2 H + \frac{\pi}{6} H^3;$$

$$\therefore \frac{h}{H} = \tfrac{3}{2} + \frac{2H^2}{D_1^2}.$$

But $$D_1^2 = 4H(D - H);$$

$$\therefore \frac{h}{H} = \tfrac{3}{2} + \frac{H}{2D - 2H} = \frac{3D - 2H}{2D - 2H}.$$

[*R. D. Bohannan.*]

EXERCISES.

70

A RIGHT ANGLE moves so that a given point in one side, distant c from the vertex, lies in a fixed axis, while the other side passes through a fixed point, distant c from this axis. Find the locus of the instantaneous centres or centrodes of the motion. [*R. H. Graves.*]

71

A LINE of unit length is bent to the arc of a circle such that the area of the segment it determines is a maximum. Find the radius of the circle and the form and area of the segment. [*O. Root, Jr.*]

72

CIRCLES of given radius are drawn through the focus of a fixed parabola, cutting the curve in four points Show that the products of the focal radii to these points are all equal. [*W. M. Thornton.*]

73

IF r is the radius of the inscribed circle of a triangle and ρ the radius of the triangle whose vertices are the feet of the altitudes of the given triangle,

$$r > 2\rho.$$

[*R. D Bohannan.*]

74

FOUR points are taken at random on the surface of a sphere. What is the probability, that all of the points do not lie in the same hemisphere?

[*A. Hall.*]

SELECTED.

75

REDUCE to its simplest form

$$\cot(x_0 - x_1)\cot(x_0 - x_2)\cot(x_0 - x_3) + \cot(x_1 - x_0)\cot(x_1 - x_2)\cot(x_1 - x_3)$$
$$- \cot(x_2 - x_0)\cot(x_2 - x_1)\cot(x_2 - x_3).$$

76

FIND the condition that the cubic

$$6x^3 - (2n + 8)x^2 + n(n + 1)x + n(n + 1)(2 - n) = 0$$

may have equal roots.

77

CONSTRUCT a square; given one vertex and two parallel lines on which the extremities of the opposite diagonal are located.

78

A, B, C are the points of application of three parallel faces x, y, z; Q is their centre; ρ is their resultant; p, q, r are the radius vectors of A, B, C measured from an arbitrary origin M. Show that if $MQ = h$

$$h^2 = \frac{p^2x + q^2y + r^2z}{\rho} - \frac{a^2yz + b^2zx + c^2xy}{\rho^2}.$$